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International Journal of Multiphase Flow 26 (2000) 83–97

International Journal of Multiphase Flow

www.elsevier.com/locate/ijmulflow

Effective thermal conductivities of a single species and a binary mixture of granular materials

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Received 1 May 1997; received in revised form 28 December 1998

Abstract

The effective thermal conductivity is developed by employing the dense-gas kinetic theory. The free path used in the theory varies with the particle velocity. The analytical results can be used for the whole range of the product of Biot number and Fourier number provided that the Biot number is less than 0.1. For the very small Biot–Fourier numbers, the conductivities are found to increase with the particle diameters and the square root of granular temperatures. For the limit of very large Biot–Fourier numbers, the effective thermal conductivity is found to be linearly proportional to the granular temperature. The effective thermal conductivities for the binary mixtures are also derived. The influences of the species concentration, the total solid fraction and the Biot–Fourier number on the thermal conductivity are investigated. Increasing the concentration of the smaller particles or reducing the size of the smaller species can increase the thermal conduction in the binary mixture system.

Keywords: Granular flow; Dense-gas kinetic theory; Granular temperature; Thermal conductivity

1. Introduction

A granular flow is a two-phase flow with an assembly of discrete solid particles dispersed in a fluid. There are many industrial applications such as the transport of ore, coal, mineral concentrate, food products or tablets. The gaseous phase plays a negligible role in the flow

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PII: S0301-9322(99)00003-8

mechanics in these dry noncohesive solid flows. Instead, collisions between the particles and between the particles and boundaries dominate the flow mechanics and result in random motions of the particles. In recent years, dense-gas kinetic theories (Chapman and Cowling, 1970) have been applied to analyze the momentum transport relations for granular material flows (Jenkins and Savage, 1983; Lun et al., 1984; Jenkins and Richman, 1985).

In granular flows, the streaming or kinetic mode, and the collisional mode are the two most important mechanisms to determine the transport properties (Campbell, 1990). The streaming or kinetic mode denotes the transport of particle properties resulting from the free motions of particles between collisions. The transfer of the properties during collisions is called the collisional mode. The application of the dense-gas kinetic theory has advanced the development of the constitutive relations for the granular material flows (Ahn and Brennen, 1991).

The transport processes in industry also include mixing or segregation of granular materials. Researchers have begun to investigate these related problems (Savage and Lun, 1988; Hsiau and Hunt, 1993a, 1996; Hunt et al., 1994; Hsiau and Yu, 1997). Besides, with many of the transport processes encountered in industry, the granular materials are also dried, heated or cooled while they are flowing (Kunii, 1980; Suzuki et al., 1980; Richard and Raghavan, 1984; Ferron and Singh, 1991, Duquenne et al., 1994). Industries depend only on empirical information, and some more general design methods need to be developed for a range of material flow rates and different types of materials (Stephens and Bridgwater, 1978; Bridgwater et al., 1985). Most of the heat transfer studies discuss convection and have many similarities to the convection in fluidized beds. The related studies include the heat transfer coefficient measurements for flows in inclined chutes (Patton et al., 1986; Ahn, 1989), and in vertical channels (Sullivan and Sabersky, 1975). However, these studies did not include the effects of particle mixing, and the kinetic theory was not employed.

The thermal conductivity is a very important property, but this property of granular material receives very little attention. Due to the difficulty in measurement, there was only one experimental study that measured the thermal conductivity of granular materials performed by Wang and Campbell (1992) in an annular shear cell. By measuring the temperature and the heat flux, the effective thermal conductivity was determined. Theoretical derivations of the thermal conductivity were also very few. By mean-free-path argument and assuming that the fluctuating velocity distribution was Maxwellian, Hunt and Hsiau (1990) developed a theory for the effective thermal conductivity of low-density granular flows. Hsiau and Hunt (1993b) analytically developed the effective thermal conductivity in a shear-induced granular material flow with a very low product of Biot number and Fourier number. The result was compared qualitatively with the experimental measurements by Wang and Campbell (1992). Hsiau (1995) followed similar reasoning, but used a more rigorous assumption to develop the analytical relation for the effective thermal conductivity. The theory is more general and can be used without the assumption of a very small Biot–Fourier number. The above analyses all used the averaged mean free path; however, the free path varies with the local particle velocity (Chapman and Cowling, 1970). The present study used the velocity-varied free path in dense-gas kinetic theory and derived the effective thermal conductivity. The two-dimensional discrete element simulations were also employed by Hunt (1997) to analyze the thermal conductivity and compared well with the theoretical results from dense-gas kinetic theory.

Most studies about granular material are for the single species. However, in real applications, the particle sizes are usually not uniform. Because of the complications involved in the transport of multicomponent mixtures, the topic receives little attention. Recently Jenkins and Mancini (1989) used revised Enskog theory to develop the kinetic theory for binary mixtures. Hsiau and Hunt (1996) then used the theory to analyze the granular thermal diffusion phenomena. Based on the dense-gas kinetic theory, the present study derived the effective thermal conductivity for the binary mixtures.

2. Single-sized granular materials

Similar to the motions of molecules in a gas, the particles in a granular material have velocities and associated properties that deviated from the mean value. Employing the dense-gas kinetic theory (Chapman and Cowling, 1970), the fluctuating velocities of the particles are assumed to follow the singlet velocity distribution function $f^{(1)}(\mathbf{C})$. Since the particle motion is not self-sustaining, the velocity distribution function is not Maxwellian. In this case, the singlet velocity distribution function is assumed to be

$$f^{(1)}(\mathbf{C}) = f^{(0)}(\mathbf{C})(1 + \Phi) \tag{1}$$

where $\mathbf{C}$ is the fluctuating velocity and Φ is a perturbation term, $\Phi \ll 1$. The Maxwellian distribution function is expressed as follows:

$$f^{(0)}(\mathbf{C}) = \frac{n}{(2\pi\Theta)^{3/2}} \exp\left(-\frac{C^2}{2\Theta}\right) \tag{2}$$

where n is the number density, C is the magnitude of the fluctuating velocity $\mathbf{C}$, and Θ is the granular temperature. Analogous to the temperature in the gases, the granular temperature quantifies the specific kinetic energy of the granular system and is defined by $\Theta = \langle C^2 \rangle / 3$. The symbol $\langle\ \rangle$ denotes the ensemble-average quantity. The ensemble average of any system property Ψ (such as mass, momentum or energy) is defined as:

$$\langle \Psi \rangle = \frac{1}{n} \int \Psi f^{(1)}(\mathbf{C})\, \mathrm{d}\mathbf{C} \tag{3}$$

where $f^{(1)}(\mathbf{C})\, \mathrm{d}\mathbf{C}$ means the probable number of particles per unit volume with a fluctuating velocity within the velocity element $\mathrm{d}\mathbf{C}$ centered at $\mathbf{C}$ and $\mathrm{d}\mathbf{C} = \mathrm{d}C_x\, \mathrm{d}C_y\, \mathrm{d}C_z$. Because of the complexity of the perturbation function Φ, it is not listed here and can be found in the papers by Lun et al. (1984) and by Hsiau and Hunt (1993b). It should be mentioned that the perturbation function Φ is an odd function of C_y.

All particles are assumed identical, spherical, smooth (no friction) and nearly elastic. All collisions in the granular flow system are assumed to be binary collisions. A pair distribution function is defined as the number of pairs of contacting particles i and j having velocities within the range $\mathbf{C}_i$ to $\mathbf{C}_i + \mathrm{d}\mathbf{C}_i$ and $\mathbf{C}_j$ to $\mathbf{C}_j + \mathrm{d}\mathbf{C}_j$ (Lun et al., 1984):

$$f^{(2)}(\mathbf{C}_i, \mathbf{C}_j) = g_0(\nu) f^{(1)}(\mathbf{C}_i) f^{(1)}(\mathbf{C}_j) \tag{4}$$

where ν is the solid fraction, and $g_0(\nu)$ denotes a correction factor which is called as the radial distribution function. An empirical form of this radial distribution function is given by Carnahan and Starling (1969) as follows:

$$g_0(\nu) = (2-\nu)/2(1-\nu)^3 \tag{5}$$

Consider a small local region and assume properties are constant in this small region. Assume that the particle has a diameter of d, a thermal conductivity of k_p, a specific heat of c_p, a total surface area of A_p, and a heat transfer coefficient between the particle and the surrounding fluid, h. Only the temperature gradient in the y direction is considered, as shown in Fig. 1. For Biot number, $Bi = hd/k_p$, less than 0.1, the lumped system analysis can be employed for the particles. In the granular material, the heat transfer resulting from the fluid motions is neglected because the heat capacity and density of the fluid are much smaller than those of particles (Hsiau and Hunt, 1993b; Hsiau, 1995; Hunt, 1997). The thermal radiation is also neglected in the present analysis. Consider a particle moving a short distance l to a new position with fluid temperature T_0. By solving the energy equation, the particle temperature at the new position can be obtained (Hsiau and Hunt, 1993b):

$$T = T_0 - C_y\sigma\frac{dT}{dy}\left[1 - \exp\left(-\frac{l_y}{\sigma C_y}\right)\right] \tag{6}$$

where $\sigma = mc_p/hA_p$, m is the particle mass, l_y is the y component of l, C_y is the y component of the particle's fluctuating velocity $\mathbf{C}$. The dimensionless group $l_y/\sigma C_y = l/\sigma C$ is the product of the Biot number and the Fourier number, where the Fourier number is $Fo = k_p t A_p/dmc_p$.

The excess energy carried by the particle to this position relative to the surrounding fluid is $\Delta e = mc_p(T - T_0)$. Using $l_y/\sigma C_y = l/\sigma C$ in Eq. (6) and assuming that the characteristic length l is the free path, λ, of the particles in the flow field, the excess energy is found as:

$$\Delta e = -mc_p\sigma C_y\left[1 - \exp\left(-\frac{\lambda}{\sigma C}\right)\right]\frac{dT}{dy} \tag{7}$$

The conduction between particles during collisions is negligible because the collisional time is

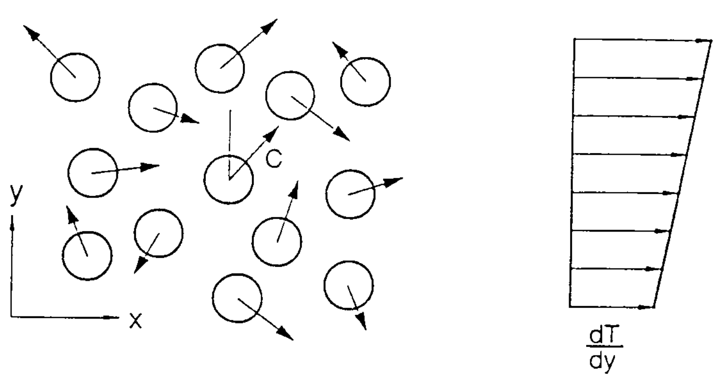

Fig. 1. Configuration for the thermal energy flux.

very short and the contact area is very small (Sun and Chen, 1988). Therefore, only the streaming mode is considered in the heat transfer. The heat flux in y direction is found by integrating the product of C_y and the excess energy carried by particles over the entire velocity space:

$$q_y = n\langle \Delta e C_y \rangle = \int \Delta e C_y f^{(1)}(\mathbf{C}) \mathrm{d}\mathbf{C} \tag{8}$$

Substituting Eqs. (1) and (7) into Eq. (8) and noting that the product of $\Delta e C_y$ and the perturbation part of $f^{(1)}$ is an odd function of C_y, the local heat flux becomes

$$q_y = -\frac{4\pi}{3} mnc_\mathrm{p} \frac{\sigma}{(2\pi\Theta)^{3/2}} \frac{\mathrm{d}T}{\mathrm{d}y} \int_0^\infty C^4 \left[1 - \exp\left(-\frac{\lambda}{\sigma C}\right)\right] \exp\left(-\frac{C^2}{2\Theta}\right) \mathrm{d}C \tag{9}$$

Treating Eq. (9), Hsiau (1995) used two methods: (1) taking the free path as the mean value, $\lambda = d/[6\sqrt{2}\nu g_0(\nu)]$; (2) taking λ/C as the averaged collisional frequency. However, in the rigorous kinetic theory, the free path is dependent on the fluctuating velocity (Chapman and Cowling, 1970). Hunt (1997) used the two-dimensional discrete element method to simulate the free paths of particles in a sheared granular flow and demonstrated that the free path was dependent on the fluctuating velocity. Using the similar mathematical derivation by Chapman and Cowling, but considering the effect of the volume occupied by particles (the solid fraction ν and the radial distribution function g_0), the free path of granular materials can be derived as:

$$\lambda(C) = \frac{w^2}{\sqrt{\pi} n d^2 E(w) g_0(\nu)} \tag{10}$$

where $w = C/\sqrt{2\Theta}$ and $E(w)$ denotes the function

$$E(w) = w\mathrm{e}^{-w^2} + (2w^2 + 1)\int_0^w \mathrm{e}^{-v^2}\, \mathrm{d}v \tag{11}$$

Then from Eq. (9), the thermal conductivity can be expressed as

$$k_\mathrm{eff} = \frac{8}{3\sqrt{\pi}B} \rho_\mathrm{p} c_\mathrm{p}\, \mathrm{d}\nu \Gamma \Theta^{1/2} \tag{12}$$

where Γ represents the dimensionless integral

$$\Gamma = \int_0^\infty w^4 \left[1 - \exp\left(-\frac{Bw}{6\sqrt{2/\pi} E(w) \nu g_0(\nu)}\right)\right] \mathrm{e}^{-w^2}\, \mathrm{d}w \tag{13}$$

and B is a dimensionless parameter: $B = \mathrm{d}/\sigma\Theta^{1/2}$, ρ_p is the particle density. Note that $B/BiFo = (d/\lambda)(C/\Theta^{1/2})$ is a finite number. Therefore B approaches 0 when $BiFo \ll 1$ and approaches to infinity when $BiFo \gg 1$.

The thermal conductivity can be non-dimensionalized by $\rho_\mathrm{p} c_\mathrm{p} d\Theta^{1/2}$ and Eq. (12) becomes:

$$\frac{k_{\mathrm{eff}}}{\rho_{\mathrm{p}} c_{\mathrm{p}} d \Theta^{1/2}} = \frac{8 \nu \Gamma}{3\sqrt{\pi} B} \tag{14}$$

For the limit of $B \ll 1 (BiFo \ll 1)$, Eq. (13) can be simplified as

$$\Gamma = \frac{B}{6\sqrt{2/\pi}} \frac{\gamma}{\nu g_0(\nu)} \tag{15}$$

where

$$\gamma = \int_0^\infty \frac{w^5 \mathrm{e}^{-w^2}}{E(w)} \, \mathrm{d}w = 0.209756 \tag{16}$$

Then the dimensionless conductivity becomes

$$\frac{k_{\mathrm{eff}}}{\rho_{\mathrm{p}} c_{\mathrm{p}} d \Theta^{1/2}} = \frac{2\sqrt{2}}{9} \frac{\gamma}{g_0(\nu)} \tag{17}$$

Comparing with Hsiau and Hunt's (1993b) derivation ($k_{\mathrm{eff}}/\rho_{\mathrm{p}} c_{\mathrm{p}} d \Theta^{1/2} = 1/[9\sqrt{\pi} g_0(\nu)]$), the current result is 5.16% higher. Since Hsiau and Hunt (1993b) under predicted the thermal conductivity measured by Wang and Campbell (1992), the current analysis shows a better accuracy.

For the very small B, the thermal conductivity is proportional to the square root of the granular temperature, as shown in Eq. (17). For the very large B, the conductivity can be simplified and found to be linearly proportional to the granular temperature:

$$k_{\mathrm{eff}} = \rho_{\mathrm{p}} c_{\mathrm{p}} d \Theta^{1/2} \nu / B = \rho c_{\mathrm{p}} \sigma \Theta \tag{18}$$

where $\rho = mn = \rho_{\mathrm{p}} \nu$ and the result is the same as that of Hunt and Hsiau's (1990) and Hsiau's (1995).

Fig. 2 shows the dimensionless thermal conductivity $k_{\mathrm{eff}}/\rho_{\mathrm{p}} c_{\mathrm{p}} d \Theta^{1/2}$ varied with the solid fraction ν for different values of B. For $B \to 0$, the thermal conductivity is found to decrease with the increase of B for the same solid fraction. Since $B \propto 1/\Theta^{1/2}$, the smaller granular temperature (larger B) results in a less random granular flow causing less thermal energy transferred. Therefore the effective thermal conductivity is smaller.

From Fig. 2, for the very small value of B (0.001), the dimensionless thermal conductivity decreases with the solid fraction. The curve for B = 0.001 is almost coincident with that for $B \to 0$. It means that the very small B approach can be used for the case of $B \leq 0.001$. As discussed above, only the streaming mode is considered in heat transfer, hence, for the flow with higher solid fraction, there are more particles transporting the thermal energy in the flowfield. However, the free path also decreases with the increase of the solid fraction, $\lambda \propto 1/[\nu g_0(\nu)]$ (see Eq. (10)). This effect reduces the heat transfer rate because of the particle collisions. For the case of smaller B (larger granular temperature), the latter factor is more important and the thermal conductivity decreases with the increase of solid fraction. The exception of the increasing trend is for a very small solid fraction (ν < 0.001), as shown in the small figure (extracted from Fig. 2 for ν < 0.01) attached to Fig. 2. For the limit case of zero

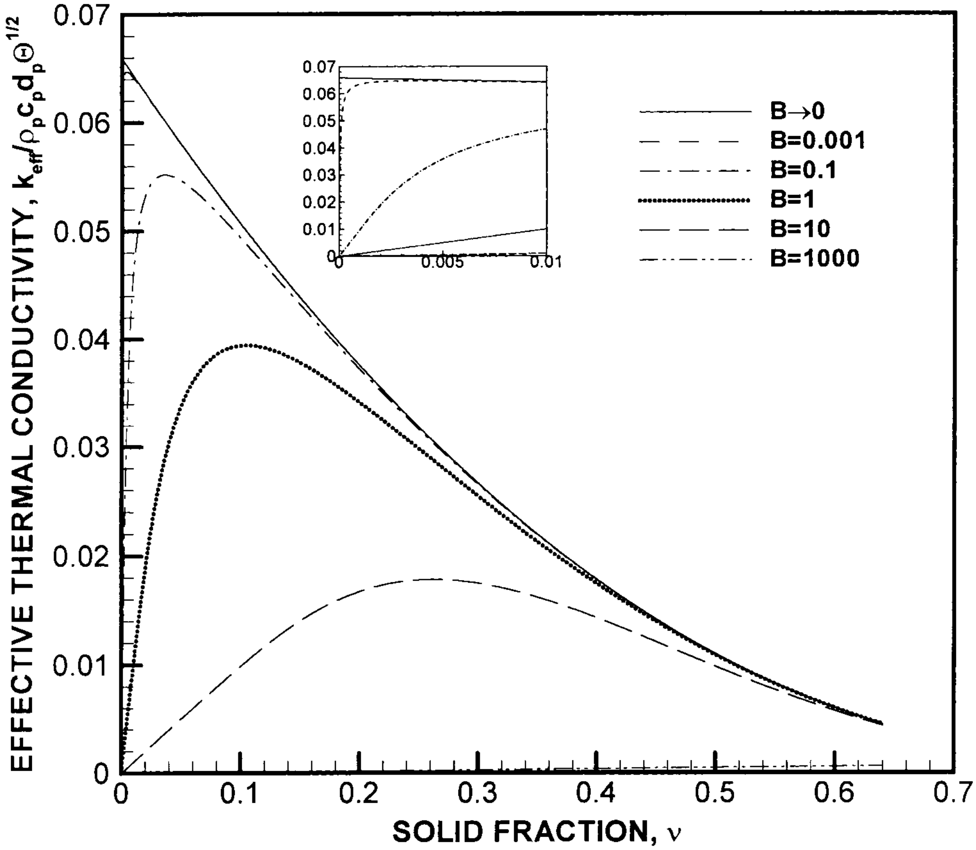

Fig. 2. The dimensionless thermal conductivity varied with solid fraction for different B values.

solid fraction ($\nu = 0$), the effective thermal conductivity should be 0 since there is no granular material. Note that the approach of $B \rightarrow 0$ is failed in this limit case.

Contrary to the small B case, for the limit of very large B, the conductivity is linearly proportional to the solid fraction (see Eq. (18)). From Eq. (7), the factor inside the bracket increases with the solid fraction rapidly for the very large values of B, i.e. the excess energy carried by each particle increases markedly with solid fraction. Therefore, the thermal conductivity increases with the solid fraction for the very large values of B (1000) as shown in Fig. 2. For the moderate values of B, the thermal conductivity first increases with the solid fraction until reaching the maximum value, and then decreases with the solid fraction. From the small figure attached in Fig. 2, the thermal conductivity should approach 0 when the solid fraction approaches 0.

3. Binary mixture of granular materials

Let the subscripts α and β represent two different species in the binary mixture, and the indices i, j are either α or β. All particles are assumed spherical, smooth and slightly inelastic.

Similar to Eq. (1), the singlet velocity distribution function is assumed to be

$$f_i^{(1)}(\mathbf{C}_i) = f_i^{(0)}(\mathbf{C}_i)(1+\Phi_i) \tag{19}$$

where Φ_i is the perturbation term with a very complicated form which can be found in Jenkins and Mancini's paper (1989) and $f_i^{(0)}(\mathbf{C}_i)$ is the Maxwellian distribution function:

$$f_i^{(0)}(\mathbf{C}_i) = \frac{n_i}{(2\pi\Theta_i)^{3/2}}\exp\left(-\frac{C_i^2}{2\Theta_i}\right) \tag{20}$$

where Θ_i is the granular temperature of species i defined by $\Theta_i \equiv \langle C_i^3 \rangle/3$. The granular temperature of the mixture is then defined by (Farrel et al., 1986; Jenkins and Mancini, 1989)

$$\Theta = \frac{1}{m_0 n}(\rho_\alpha\Theta_\alpha + \rho_\beta\Theta_\beta) \tag{21}$$

where n is the total density, $n = n_\alpha + \mathrm{n}_\beta$, ρ_α and ρ_β are the bulk densities for the two species in the flows, $\rho_i = \rho_{\mathrm{p}i}\nu_i = m_i n_i$, and $m_0 = m_\alpha + m_\beta$. For a binary mixture, the equipartition of fluctuating energy is assumed (Shen, 1984; Farrel et al., 1986; Jenkins and Mancini, 1989):

$$\frac{3}{2}m_\alpha\Theta_\alpha = \frac{3}{2}m_\beta\Theta_\beta \tag{22}$$

Then the Maxwellian distribution function for the ith species is rewritten as

$$f_i^{(0)}(\mathbf{C}_i) = n_i\left(\frac{m_i}{2\pi m_0\Theta_i}\right)^{3/2}\exp\left(-\frac{m_i C_i^2}{2m_0\Theta_i}\right) \tag{23}$$

The pair-distribution function for particles i and j is defined as

$$f_{ij}^{(2)}(\mathbf{C}_i,\mathbf{C}_j) = g_{ij} f_i^{(1)}(\mathbf{C}_i) f_i^{(1)}(\mathbf{C}_j) \tag{24}$$

where g_{ij} is the radial distribution function of two particles evaluated when particles are in contact and is expressed as (Kincaid et al., 1983; Jenkins and Mancini, 1989):

$$g_{ij} = \left[Z^2 + \frac{3d_i d_j}{d_i+d_j}ZZ_2 + 2\left(\frac{d_i d_j}{d_i+d_j}\right)^2 Z_2^2\right]/Z^3 \tag{25}$$

where

$$Z_l = \frac{\pi}{6}\sum_{j=\alpha,\beta} n_j d_j^l$$

$$l = 1,2,3$$

$$Z = 1 - Z_3 \tag{26}$$

Since the perturbation term in Eq. (19) is very small, it is neglected in the present study to simplify calculations.

For heat transfer, only the streaming mode is considered. Then the heat flux in the y direction can be found as:

$$q_y = n_\alpha \langle \Delta e_\alpha C_{\alpha y} \rangle + n_\beta \langle \Delta e_\beta C_{\beta y} \rangle \tag{27}$$

where Δe_i denotes the excess energy carried by particle i:

$$\Delta e_i = -m_i c_{\mathrm{p}i} \sigma_i C_{iy} \left[1 - \exp\left(-\frac{\lambda_i}{\sigma_i C_i} \right) \right] \frac{\mathrm{d}T}{\mathrm{d}y} \tag{28}$$

The free path of species i in Eq. (28) is denoted by λ_i which can be derived from the dense-gas kinetic theory (Chapman and Cowling, 1970),

$$\lambda_i = \frac{w_i^2 / E(w_i)}{n_i d_i^2 \sqrt{\pi} g_{ii} + n_j d_{ij}^2 \sqrt{\pi} \sqrt{m_i/m_j} g_{ij}} \quad j \neq i \tag{29}$$

where $w_i = C_i \sqrt{(m_i/2m_0 \Theta)}$ and $d_{ij} = (d_i + d_j)/2$.

Using the dimensionless parameter $B_i = d_i/\sigma_i \Theta_i^{1/2}$, the term $\lambda_i/\sigma_i C_i$ in Eq. (28) can be expressed as

$$\frac{\lambda_i}{\sigma_i C_i} = w_i \alpha_i B_i \tag{30}$$

where

$$\alpha_i = \frac{1}{\sqrt{2\pi} E(w_i) [n_i d_i^3 g_{ii} + n_j d_i d_{ij}^2 \sqrt{m_i/m_j} g_{ij}]} \quad j \neq i \tag{31}$$

Substituting Eqs. (28) and (30) into Eq. (27), the effective thermal conductivity is found as:

$$k_{\mathrm{eff}} = \frac{8}{3\sqrt{\pi}} \sum_{i=\alpha,\beta} \frac{m_0}{m_i B_i} \rho_{\mathrm{p}i} c_{\mathrm{p}i} d_i \nu_i \Gamma_i \Theta^{1/2} \tag{32}$$

where Γ_i denotes the integral

$$\Gamma_i = \int_0^\infty w_i^4 [1 - \mathrm{e}^{w_i \alpha_i B_i}] \mathrm{e}^{-w_i^2} \, \mathrm{d}w_i \tag{33}$$

If $\alpha = \beta$, Eq. (32) can be simplified to Eq. (12) (except the different representations of the radial distribution functions and noting $\Theta = \Theta_\alpha/2 = \Theta_\beta\ /2$) which is for the single-sized granular materials.

For $(BiFo)_i \ll 1$, i.e. $B_i \ll 1$, the term $1 - \mathrm{e}^{w_i} \alpha_i B_i$ approaches $w_i \alpha_i B_i$ and Eq. (32) is simplified to

$$k_{\mathrm{eff}} = \frac{8}{3\sqrt{\pi}} \sum_{i=\alpha,\beta} \frac{m_0}{m_i} \rho_{\mathrm{p}i} c_{\mathrm{p}i} d_i \nu_i \gamma_i \Theta^{1/2} \tag{34}$$

where

$$\gamma_i = \int_0^\infty \alpha_i w_i^5 e^{-w_i^2} dw_i \tag{35}$$

The thermal conductivity for this case is proportional to the square-root of the granular temperature.

For $(BiFo)_i \gg 1$, i.e. $B_i \gg 1$, from Eq. (33), the value of Γ_i is found to be $3\sqrt{\pi}/8$. Then the thermal conductivity in Eq. (32) can be simplified to

$$k_{\rm eff} = \sum_{i=\alpha,\beta} \frac{m_0}{m_i} \rho_i c_{\rm pi} \sigma_i \Theta \tag{36}$$

The thermal conductivity for this case is linearly proportional to the granular temperature. Assuming that the two types of particles are of the same material, i.e. $\rho_{\rm p\alpha} = \rho_{\rm p\beta} = \rho_{\rm p}$, $c_{\rm p\alpha} = c_{\rm p\beta} = c_{\rm p}$, then the thermal conductivity can be non-dimensionalized as

$$\frac{k_{\rm eff}}{\rho_{\rm p} c_{\rm p} d_\alpha \Theta} = \frac{8}{3\sqrt{\pi}} \left[\frac{[1 + (d_\beta/d_\alpha)^3]\nu_\alpha \Gamma_\alpha}{B_\alpha} + \frac{d_\beta}{d_\alpha} \frac{[1 + (d_\alpha/d_\beta)^3]\nu_\beta \Gamma_\beta}{B_\beta} \right] \tag{37}$$

If $d_\alpha = 2\, d_\beta$, by the definitions of B_i and σ_i, Eq. (22), and the heat transfer correlation for flows passing a sphere (Incropera and De Witt, 1990), the following relation is derived: $B_\alpha = 2B_\beta$. Using $B_\alpha = 20$ and $B_\beta = 10$, Eq. (37) can be calculated for different values of ν_α/ν, and the results are plotted in Fig. 3. Since the B_i values are moderate, there exists a maximum value of the effective thermal conductivity, as discussed in last section (see Fig. 2). It is found that the greater the number of smaller particles (lower ν_α/ν value), the higher the effective thermal conductivity. Note that from Eq. (12), the conductivity is proportional to the product of the particle diameter and the square root of granular temperature for a single-sized material. In a binary mixture, the granular temperature of the smaller particles is greater than that of the larger particles (see Eq. (22)). When the number of smaller particles increases (ν_α/ν decreases), the product of $d_i\Theta_i^{1/2}$ is greater for the smaller species which results in the increase of the overall thermal conductivity of the system (see Eq. (34)).

Fig. 4 shows the effective thermal conductivity of the binary mixture ($k_{\rm eff}$) varied with ν_α/ν for $B_\alpha = 20$, $B_\beta = 10$, and total solid fractions of 0.1, 0.2, 0.3, 0.4, 0.5 and 0.6. The effective thermal conductivity of the binary mixture $k_{\rm eff}$ is non-dimensionalized by the thermal conductivity of the single species β, $(k_{\rm eff})_\beta$. If the system is not too dense ($\nu < 0.5$), the thermal conductivity decreases when the number of larger particles increases (greater ν_α/ν value). This information can also found from Fig. 3, but for a very dense system, the thermal conductivity increases very slowly with the increase of ν_α/ν. Until reaching the maximum value, the thermal conductivity decreases rapidly with the increase of ν_α/ν.

With $\nu_\alpha/\nu = 0.5$ and $B_\alpha = 20$, Fig. 5 shows the effective thermal conductivity plotted against the total solid fraction for different values of B_β. The trends of curves are similar to those in Fig. 3. For the smaller B_β, the maximum values of thermal conductivity occurs at the lower total solid fraction. This phenomena is also shown in Fig. 2 for the single species materials. From Fig. 5, the effective thermal conductivity increases with the decrease of B_β (smaller B_β

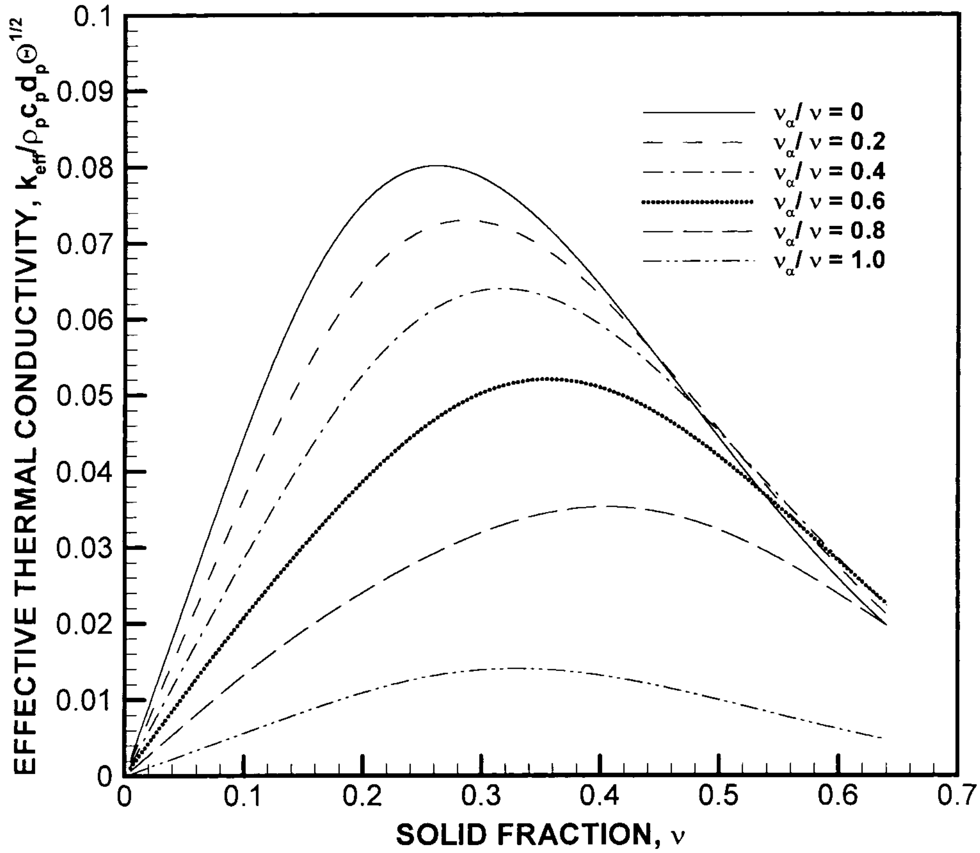

Fig. 3. The dimensionless thermal conductivity for a binary mixture varied with total solid fraction, $B_\alpha = 20$, $B_\beta = 10$.

denotes smaller size of the smaller particles). Therefore, increasing the concentration or reducing the size of the smaller species can cause the increase of thermal conduction in the system.

4. Conclusions

The dense-gas kinetic theory is employed to investigate the thermal conduction in a granular flow. The concept of velocity-dependent free path is introduced in the derivations. The analytical results can be used for the whole range of the product of Biot and Fourier numbers if the Biot number is less than 0.1. For the system with a very small *BiFo* value, the thermal conductivity is proportional to the square-root of the granular temperature and decreases with the increase of the solid fraction. The current result is more accurate than Hsiau and Hunt's (1993b). For the very large *BiFo* values, the conductivity is linearly proportional to the

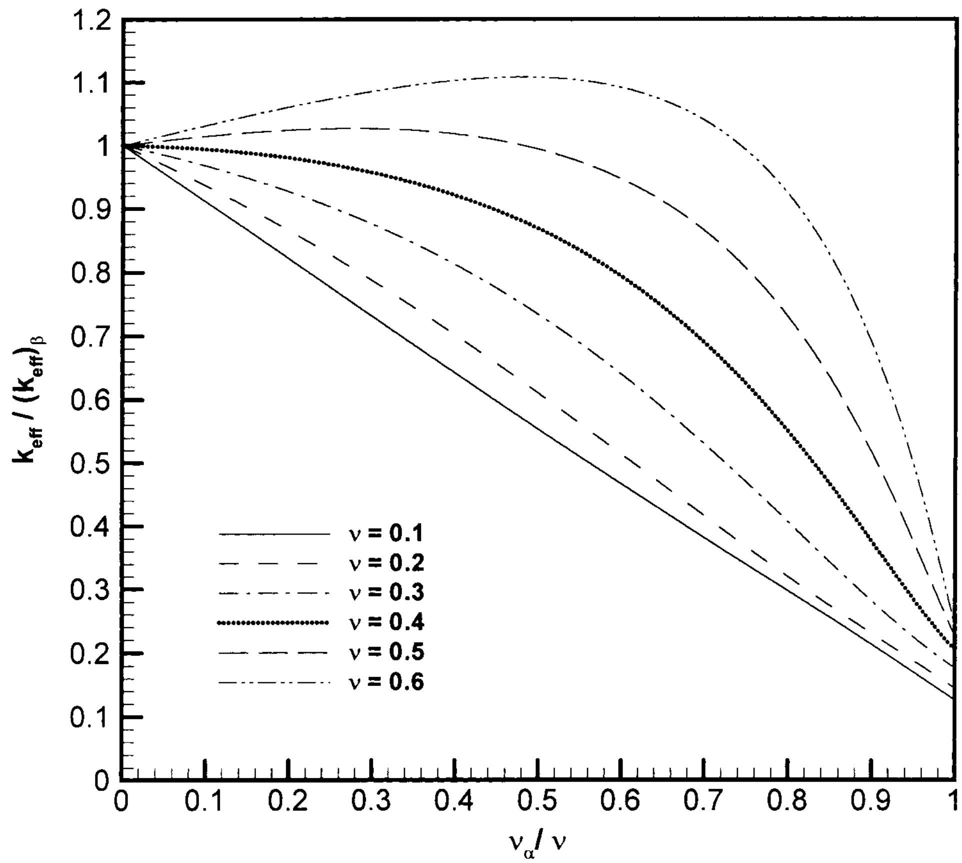

Fig. 4. The effective thermal conductivity ($k_{\rm eff}/(k_{\rm eff})_\beta$) varied with ν_α/ν for different total solid fractions, $B_\alpha = 20$, $B_\beta = 10$.

granular temperature and the solid fraction. For the moderate *BiFo* values, the conductivity increases with the solid fraction when the solid fraction is small and reaches the maximum value, then starts to drop with the increase of solid fraction. The thermal conductivity for the binary mixture system is also derived. For binary-sized systems with the same total solid fraction, the higher concentration of the smaller species results in the greater thermal conductivity provided that the system is not very dense. Reducing the size of smaller particles is also helpful in increasing the thermal conductivity.

Due to the difficulty in the measurements in granular systems, there are still no appropriate experimental data available for comparison. However, the current study provides an analytical model to evaluate the effective thermal conductivities in single-sized and binary mixture of granular materials, which should be very helpful in the development of the research field about granular flows, and can also provide more design information for the related industries. The next important subject for researchers is to develop the experimental technique to measure the thermal properties in granular flows.

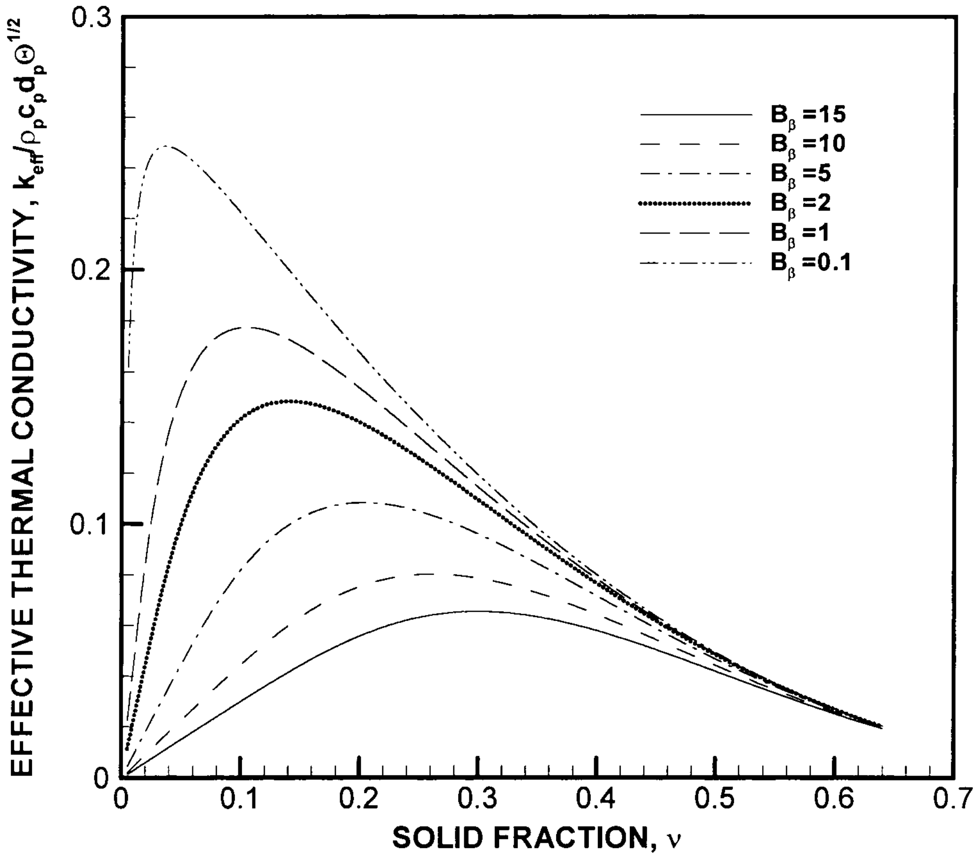

Fig. 5. The dimensionless thermal conductivity plotted against the total solid fraction for different values of B_β, $B_\alpha = 20$, $\nu_\alpha/\nu = 0.5$.

Acknowledgements

The author wishes to acknowledge the support from the National Science Council of the ROC. for this work through grant NSC 83-0410-E-008-018.

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